

## Grosvenor Road

## Primary School

# Calculations Policy 

Incl. Fractions Policy and KS2 Vocabulary List

This policy outlines how calculations are to be taught at Grosvenor Road Primary School. Teachers should use it alongside the Mathematics policy to deliver a consistent approach towards calculations. The policy aims to assist teachers in delivering a programme geared towards mastery of Mathematics in line with The National Curriculum (DfE 2014)

## Our Aim at Grosvenor Road

Each child should be able to think and solve problems mathematically by using the appropriate skills, concepts and knowledge. They should be provided with rich and enjoyable experiences related both to their individual needs and to the wider requirements of society.

We aim for each child to:
I. Be Positive - they develop a curious, inquisitive and positive approach to Mathematics as a subject- a 'can do attitude'.
2. Be Fluent - over time they have a conceptual understanding of mathematics and the can recall key facts easily.
3. Solve Problems - they can apply skills to unfamiliar problems, they can break a problem into key parts and they persevere to find a solution.

Taken from the school's mathematics policy

As a school, we aim to use this policy to develop children who are numerate thinkers and that have a concrete understanding of the fundamentals of Mathematics. Children are taught using shared language and a consistent approach towards resources, images and models. The following resources are used throughout the school in order to underpin a child's learning:



## Early learning Goal:

Have a deep understanding of number to 10, including the composition of each number: subitise up to 5; automatically recall number bonds to 5 and some number bonds to 10 , including double facts.

Children should be encouraged to develop their own mental picture surrounding numbers in preparation for calculation. They should be given a wide range of opportunities to experiment with Addition in practical scenarios, e.g. role play, counting games and small world play. Children should be given access to a wide range of mathematical resources for counting so that they are aware of them and how they can be used later in the school. Children should begin to identify their own mathematical problems based on their own interest and fascinations using the resources and equipment made available to them.

## Counting all method

Children will begin to develop their ability to add by using practical equipment to count out the correct amount for each number in the calculation and then combine them to find the total.

## Counting on method

To support children in moving from a counting all strategy to one involving counting on, children should still have two groups of objects but one should be covered so that it cannot be counted, children should be encouraged to use their knowledge of the order of numbers to count on mentally without the visual stimulus of the second group.

In Nursery, children are introduced to a sense of number through Numicon. This should be further developed in Reception: using the Firm Foundations Numicon kit.


On top of this, children in Nursery also develop their number skills and confidence through 'Let's visit Numberland.' This scheme gives the children the opportunity to underpin their knowledge of number through practical activities, songs and free play, which helps to give a context for problem solving at an early age. This should continue to be developed in reception using the 'Let's visit Numberland' handbook.


## End of year objective:

Add $\mathbf{1}$ and $\mathbf{2}$ digit numbers up to $\mathbf{2 0}$ using concrete objects and pictorial representations.

Children should continue to use a wide range of mathematical apparatus and jottings to approach Addition. They should combine separate groups of objects to find a total using the counting all or counting on method initially approached in EY. Children should also continue to use Numicon Kit 1 to underpin mental methods and their understanding of number. For further in depth guidance on the use of Numicon, please consult the Numicon Handbook.

$4+3=7$

$2+\ldots=5$

Alongside Numicon, the children should also be introduced to Base 10 equipment in preparation for the move to formal Addition and place value.

The children can make numbers using the Base 10 equipment using a mixture of 'ten rods' and 'ones cubes' to create numbers greater than 10 but less than 20.


Here we can see the number 12 made up of a ten rod and 2 one cubes. This can then be added to as the support for the calculation of an Addition problem.


Above, the child has added further one cubes to the initial number of twelve to create 18. In this case the child could group the one cubes together to aid with counting. Where possible the child should be encouraged to record this problem as $12+6=18$ using the correct mathematical symbols.

## Children should also be able to:

- Read, write and understand Addition problems using the Addition (+) and Equals (=) symbols.
- Solve one step Addition problems including missing number problems.
- Represent and use number bonds to 20.


## End of year objective:

Add numbers using concrete objects, pictorial representations, and mentally, including: a 2 digit number and units; a $\mathbf{2}$ digit number and tens; two $\mathbf{2}$ digit numbers; three $\mathbf{1}$ digit numbers.

Children should continue to use Base 10 equipment to support their understanding of mental and written Addition.
E.G. $24+4=28$ Initially, the children should approach an Addition problem using concrete objects, in this case Base 10, they should create each number in the calculation starting with the tens and then moving on to any units.

## Concrete

 objects:
-7


When drawing a pictorial representation of the calculation using Base 10 apparatus, the children should be taught and encouraged to represent a 10 rod using a slanted line and dots for the ones blocks. Underneath each pictorial representation the children should be encouraged to write the value of each diagram to familiarise themselves with written calculations.

The children should be taught to recognise that Addition is commutative and that the numbers can be rearranged within the calculation.

| : |  |  |  | // : : |
| :---: | :---: | :---: | :---: | :---: |
| 4 | $\pm$ | 24 | = | 28 |
|  |  | : |  | $/ /::$ |
| 24 | $\pm$ | 4 | $=$ | 28 |

The children should continue to use this method when exchanging is required. When the units total more than 10 , the children should be taught and encouraged to exchange 10 ones cubes for a 10 rod in preparation for exchanging within Formal Columnar Addition. For example, in the calculation $36+15=51$, the children would start by creating each number using the Base 10 apparatus.


6 ones blocks + 5 ones blocks $=11$ ones blocks. Within this calculation the children should recognise that 11 one blocks can be exchanged for one 10 rod and a single one block


So using concrete representation the calculation should look like:



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As the children become more confident with Addition they should begin to layout the written calculation using the formal method alongside the pictorial representation in preparation for the move to Formal Columnar Addition in Y3.


The children should be taught and encouraged to set the digits out into their place value columns writing the total of the calculation in between the two total lines.

## Children should also be able to:

- Use concrete and pictorial resources to solve Addition problems in context and missing number problems.
- Use inverse operations to solve problems and check the accuracy of an answer.
- Recall and use Addition and Subtraction facts to 20 fluently, and derive and use related facts up to 100.


## End of year objective:

Add numbers using formal written methods (Columnar) with numbers up to and including $\mathbf{3}$ digits.
Children should build upon their knowledge of Addition from Y2 and move towards formal written Columnar Addition with numbers up to and including 3 digits. Initially, children should work on numbers that do not require exchanging to cement the skill and to re-familiarise themselves with the layout.

Children should be taught and encouraged to recognise

| 4 | 7 |
| ---: | :--- |
| $+\quad 5$ | 1 |
| 9 | 8 | the place value of each digit, setting out each number into its place value columns.

When setting out the Columnar Addition calculation, children should be taught to avoid leaving a gap between their total line and the last row of digits in preparation for exchanging under the total line.
exchanging, children should be taught to apply the method to 2 digit numbers that require them to exchange digits.


> In this calculation, the children should recognise that $7+5=12$. They should be taught to place the one from 12,2, within the total lines and to 'exchange' the 1 , representing 10 , under the total line and in the tens column. The tens column calculation then becomes $3+5+1=9$ representing $30+50+10=90$.

Once the children have become confident with the layout and conventions of Columnar Addition with 2 digit numbers and exchanging, they should introduce the skill to 3 digit numbers. Children should be taught and encouraged to recognise that it is not always a 'ten' that is exchanged.


## Children should also be able to:

- Add numbers mentally including: a three digit number and a 1 digit number; a 3 digit number and a 2 digit number and a 3 digit number and a 3 digit number.
- Estimate and use inverse operations to check the accuracy of an answer.
- Solve a range of Addition problems including missing number problems.


## End of year objective:

Use formal written methods (Columnar) to add multiple whole numbers with up to 4 digits.

Children should continue to use and build on formal Addition skills that have been learnt in Y 3 and apply these skills to increasingly larger numbers and calculations that involve multiple numbers of varying sizes.

|  | 4 | 1 | 7 | 9 |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 2 | 3 | 3 |  |
| + | 1 | 4 | 3 | 9 |  |

Children should be taught and encouraged to exchange digits underneath the total line and should be given the opportunity to calculate using numbers that require them to exchange digits greater than 1.

## Children should also be able to:

- Use estimation, inverse operations and rounding to check the accuracy of an answer, both in and out of context.
- Solve two step Addition and Subtraction problems in context, deciding which operation to use and when.
- Solve simple measure and money problems involving fractions and decimals to 2 decimal places.

Liam, Sarah and Amy buy lunch at a salad bar.
Circle three numbers that add to make 750 .

| salad bar |  |  |  |
| :--- | ---: | :--- | :--- |
| Salads |  | Desserts |  |
| cheese | $£ 1.20$ | banana | $25 p$ |
| egg | 90 p | apple pie | 50 p |
| tuna | $£ 1.60$ | yogurt | 35 p |

Liam has $£ 2.50$ to spend.
He buys a tuna salad and an apple pie
How much money has he got left?

## End of year objective:

Use formal written methods (columnar) to add multiple digits ranging in size from 2 d.p. to 5 digit whole numbers.

Building on what has been taught in $Y 4$, children will continue to use and apply their knowledge of Columnar Addition to increasingly larger numbers and decimals.

| 3 | 4 | 1 | 2 | 9 |
| ---: | ---: | ---: | ---: | ---: |
|  |  | 4 | 3 | 8 |
| + | 3 | 2 | 5 | 9 |
| 3 | 7 | 8 | 2 | 6 |
|  |  | 1 | 2 |  |

Children should be able to add multiple large numbers in calculations that require them to exchange digits of differing values.

When adding decimals, children should solve calculations that
 require them to exchange digits across the decimal point.

## Children should also be able to:

- Use rounding and estimation to check the accuracy of an answer within the context of a problem.
- Solve multi-step problems in context, choosing which operation to use and when.
"Pupils should go beyond the measurement and money models of decimals, for example, by solving puzzles involving decimals."

DfE - The 2014 Primary National Curriculum - Notes and Guidance
Pupils should be given the opportunity to use their problem solving and Addition skills in regards to decimals when looking at 'pure' problem solving situations outside of context.
E.G.


## End of year objective:

To confidently use a formal written methods (Columnar addition) to add multiple decimals and whole numbers.

As part of the National Curriculum, children should be confident with Columnar Addition using digits of varying sizes, from 8 digit whole numbers to decimals with 3 decimal places.

Children should be able to add

| 8 | 2 | 3 | 9 | 2 | 4 | 7 | 8 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
|  |  |  | 9 | 3 | 2 | 1 | 0 |
| + | 1 | 2 | 9 | 2 | 2 | 2 | 1 |
| 8 | 3 | 7 | 7 | 7 | 9 | 0 | 9 | multiple large numbers requiring them to 'exchange' digits of varying values.


| 2 | 3 | . | 3 | 6 | 1 |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | 9 | . | 0 | 8 | 0 |
| 5 | 9 | . | 7 | 7 | 0 |
| + | 1 | . | 3 | 0 | 0 |
| 9 | 3 | . | 5 | 1 | 1 |
| 2 | 1 |  | 2 |  |  |

Children should continue to use their knowledge of place value to set out numbers of varying sizes, when doing this, children should be taught and encouraged to write the numbers in the same number of decimal places through identification. For example, 3 tenths is the same as 300 thousandths and thus 0.3 is the same value as 0.300 .

## Children should also be able to:

- Apply written Addition skills when solving multi-step problems in context.
E.G.
These are some prices in a fish and chip shop.

| Fish $£ 2.30$ | Peas 35 p |
| :--- | :--- |
| Sausage $£ 1.80$ | Curry sauce 40 p |
| Chips (small bag) 60 p | Bread roll 30 p |
| Chips (large bag) 90 p | Pickled onion 28 p |

Megan buys a sausage and a bread roll.
Chen buys a small bag of chips and a curry sauce.
How much more does Megan pay than Chen?

- Identify missing digits within Columnar Addition problems.




## Early learning Goal:

## Have a deep understanding of number to 10, including the composition of each number: automatically recall number bonds to $\mathbf{5}$, including subtraction facts.

Children should be encouraged to develop their own mental picture surrounding numbers in preparation for calculation. They should be given a wide range of opportunities to experiment with Subtraction in practical scenarios, e.g. role play, counting games and small world play. Children should be given access to a wide range of mathematical resources for counting so that they are aware of them and how they can be used later in the school. Children should begin to identify their own mathematical problems based on their own interest and fascinations using the resources and equipment made available to them.

## Taking away

Children should begin to develop their own mental understanding of Subtraction and should be supported with apparatus when doing so. They should begin to use a wide range of mathematical apparatus and resources to subtract two single digit numbers.

## E.G. <br> $$
7-3=4
$$



Children should start by counting out the initial number of counters before removing or covering the second number of counters. They should then solve the calculation by counting the remaining number of counters.


As the children become more confident with number and counting they should be encouraged to count back to find their answer. In this case starting at 7 and counting back 3 numbers, arriving at the answer of 4.

Further information and guidance for the teaching of Subtraction should be taken from the Numicon Firm Foundations Teaching Guide.


## End of year objective:

Subtract 1 and $\mathbf{2}$ digit numbers up to $\mathbf{2 0}$ using concrete objects and pictorial representations.

Children should continue to use practical apparatus and mathematical resources when approaching Subtraction. They should build upon taking away skills learnt in Reception and begin to record calculations using a range of jottings as well as using the appropriate mathematical symbols in more formal recordings. Children should also continue to use Numicon Kit 1 to underpin mental methods and their understanding of number. For further in depth guidance on the use of Numicon, please consult the Numicon Handbook.


Alongside Numicon, the children should continue to use Base 10 equipment in preparation for a move to formal calculations. Initially, the children should focus on using groupings of one cubes for Subtraction to avoid having to exchange. As confidence and accuracy with Subtraction grows, the children may begin to approach exchanging ten rods for one cubes with calculations.


Above we can see the child has counted out the initial number of 12 before removing the 4 counters as is stated in the written calculation. The children should be taught and encouraged to 'count away' the counters they are removing and then count the remaining counters in order to increase accuracy.

Children should also be able to:

- Read, write and understand Subtraction problems using the Subtraction (-) and Equals (=) symbols.
- Solve one step Subtraction problems including missing number problems.
- Represent and use Subtraction facts to 20.


## End of year objective:

Subtract numbers using concrete objects, pictorial representations, and mentally, including: a 2 digit number and units; a $\mathbf{2}$ digit number and tens; two $\mathbf{2}$ digit numbers; three $\mathbf{1}$ digit numbers.

Children will begin to use the Base 10 equipment to support their calculations, still using a take away or removal method. They need to understand that the number being subtracted does not appear as an amount on its own, but rather as part of the larger amount.

For example, to calculate 54-23, children would count out 54 using the Base 10 equipment (5 tens and 4 units). They need to consider whether there are enough units to remove 3 , in this case there are, so they would remove 3 units and then two tens, counting up the answer of 3 tens and 1 one to give 31.


Circling the remaining ten rods and one dots will allow the children to easily identify the final answer to their problem which will help when writing out the numerical calculation underneath.


Children should be taught and encouraged to include the written problem underneath their pictorial representation to further reinforce the value of each diagram. This will also help with the transition into written calculations.

When the number of units to be subtracted is greater than the number of units in the initial number the children must be taught and encouraged to recognise that they need to exchange a ten rod for 10 one cubes in order for the calculation to be completed successfully. For example, $54-26=28$. As with all Subtractions the children would start by counting out 54 using the Base 10 apparatus.


Children should also be able to:

- Use concrete and pictorial resources to solve Subtraction problems in context and missing number problems.
- Use inverse operations to solve problems and check the accuracy of an answer.
- Recall and use Addition and Subtraction facts to 20 fluently, and derive and use related facts up to 100.


## End of year objective:

Subtract numbers using formal written methods (Columnar) with numbers up to and including 3 digits.

Children should continue to build upon mental methods for Subtraction learnt in Y 2 throughout Y 3 , alongside this the children should be taught and encouraged to use Formal Written Subtraction, Columnar, as a means for calculating with larger numbers.

In this calculation we can see that the initial stage should be
$6-9$. Within Column Subtraction we are unable to include
negative numbers as part of our answer so we need to
exchange a ten from the next column for 10 units. The
calculation then becomes $16-9=7$. As we have exchanged 10
units from the 8, representing 80 , in the tens column this
number is now 10 units lower and thus the 8 becomes 7 in the
calculation.

As the children become more fluent with the calculation and its layout, they should begin to calculate using increasingly larger numbers and numbers of differing digit lengths using their place value skills to set the digits out into the correct columns to avoid miscalculation.


## Children should also be able to:

- Subtract numbers mentally including: a 3 digit number and a 1 digit number; a 3 digit number and a 2 digit number and a 3 digit number and a 3 digit number.
- Estimate and use inverse operations to check the accuracy of an answer.
- Solve a range of Subtraction problems including missing number problems.


## End of year objective:

Use formal written methods (Columnar) to subtract numbers with up to 4 digits.

Children should continue to use and build on formal Subtraction skills learnt in $Y 3$ and apply these to numbers with up to 4 digits. Children should also be able to subtract numbers of differing digits.


## Children should also be able to:

- Use estimation, inverse operations and rounding to check the accuracy of an answer, both in and out of context.
- Solve two step Addition and Subtraction problems in context, deciding which operation to use and when.
- Solve simple measure and money problems involving fractions and decimals to 2 decimal places.



## End of year objective:

To subtract numbers with more than 4 digits, including numbers to $2 \mathrm{~d} . \mathrm{p} .$, using formal written methods (Columnar).

Building on what has been taught in Y 4 , children should continue to use and apply their use of Columnar Subtraction to increasingly larger numbers and decimals.

When subtracting numbers of increasingly larger sizes, children will be expected to exchange across multiple columns.


Children should be able to subtract numbers of differing lengths and align the digits in the correct place value columns.

## Children should also be able to:

- Solve multistep Subtraction problems in context and decide what operation to use when.
- Use rounding and estimation to check the accuracy of an answer.
"Pupils should go beyond the measurement and money models of decimals, for example, by solving puzzles involving decimals."
- DfE - The 2014 Primary National Curriculum - Notes and Guidance

Children should again be given the opportunity to use problem solving skills involving the Subtraction of decimals outside of a context as stated within the Addition guidance.

## End of year objective:

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To subtract a range of whole and decimal numbers confidently using formal written methods (Columnar Subtraction).
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Children should build on their use of decomposition and apply this to use with digits ranging from numbers with three decimal places to 8 digit whole numbers.


When subtracting numbers of differing decimal places, children should be taught that they are the same through identification and that 2 tenths is the same as 20 hundredths, therefore 0.2 is the same value as 0.20 .

When subtracting increasingly larger numbers children will be expected to exchange across multiple columns.


## Children should also be able to:

- Solve multistep Subtraction problems in context and when combined with other operations.
E.G.

An iced cake costs 10 p more than a plain cake.
Sarah bought two of each cake.


They cost $£ 1$ altogether.
What is the cost of an iced cake?


## Early learning Goal:

## Explore and represent patterns with numbers up to 10, including double facts.

Children should be encouraged to develop their own mental picture surrounding numbers in preparation for calculation. They should be given a wide range of opportunities to experiment with Multiplication and grouping in practical scenarios, eg role play, counting games and small world play. Children should be given access to a wide range of mathematical resources for counting so that they are aware of them and how they can be used later in the school. Children should begin to identify their own mathematical problems based on their own interest and fascinations using the resources and equipment made available to them.

Children should also be given the opportunity to experiment with various 'real world' objects that show Multiplication and grouping through the use of non-formal groupings and arrays.


Children should also be encouraged to develop their own ways of recording 'real world' Multiplication through drawings and jottings.


Y1

## End of year objective:

Solve one-step problems involving Multiplication, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

Children should continue to use a wide range of practical equipment as a mean of solving Multiplication problems with the support of the teacher. They should use a wide variety of 'real world' objects that show Multiplication as an array, such as egg boxes, ice cube trays, cake tins and chocolate bars. In Year One, children should begin to approach one step problems using concrete resources and pictorial representations to underpin their learning.

## Abi has 2 pots.

She plants 7 seeds in each pot.


How many seeds does she plant altogether?


Sita puts 2 shoes in each of these boxes.
How many shoes are there altogether?

Children should also continue to use Base 10 and Numicon equipment. Further in depth guidance can be found in the Numicon Kit 1 Handbook.


End of year objective:
Calculate mathematical statements for Multiplication (using repeated Addition) and write them using the Multiplication ( x ) and Equals (=) signs.

Children should be taught to calculate Multiplication through repeated Addition supported by the use of practical apparatus, e.g. Base 10, Numicon, counters or cubes etc. Children should initially approach Multiplication at this stage through the 2,5 and 10 Multiplication Tables.
$5 \times 4=20$ can be shown using 5 equal groups of 4 counters. Initially the children should begin to group the counters together in to random groupings as below.


## $4+4+4+4+4=20$

As children become more confident with Multiplication they should be encouraged to arrange the counters, or other apparatus in a more organised pattern, as shown below. This will help with the transition into use of arrays.


Children should continue to use and adapt this method of Multiplication into the use of arrays, this helps to reinforce the commutative nature of Multiplication and will help with the transition into Formal Multiplication.

$4+4+4+4+4$

$5+5+5+5=20$

Throughout Y 2 children should be encouraged to use a range of apparatus and jottings when multiplying numbers in order to improve understanding and fluency with calculations.

## $5 \times 4=20$

## $4 \times 5=20$

Children should be taught and encouraged to use mathematical symbols when writing out the numerical problem of each Multiplication calculation.

Children should also be taught and encouraged to recognise that Multiplication is commutative, and thus the Multiplier and the Multiplicand are interchangeable.

## Children should also be able to:

- Solve problems involving Multiplication and Division, using materials, arrays, repeated Addition, mental methods, and Multiplication and Division facts, including problems in contexts.
- Recall and use Multiplication and Division facts for the 2,5 and 10 Multiplication Tables, including recognising odd and even numbers


## End of year objective:

To practise and recall mental methods for Multiplication, progressing towards formal written methods for Multiplication (Short Multiplication).

Children should continue to work towards a formal method for Multiplication throughout Y3. They should build upon knowledge of repeated Addition and Multiplication Tables facts from Y2 and continue to use concrete objects and apparatus to underpin their learning.

## Stage 1

Children should be introduced to the 3, 4 and 8 Multiplication Tables and learn through use of concrete objects and arrays that these can be displayed and written in many different ways. Children should be taught and encouraged to recognise that Multiplication facts are Commutative.
E.G.


## $3 \times 6=18$



## $6 \times 3=18$

As the children become more confident, they should be taught to recognise derived facts.
E.G. $6 \times 3=18$ so $60 \times 3=180$ and $600 \times 31800$ etc.
$60 \times 3=180$

60 is 10 times bigger than 6, so I know that my answer must be at least 10 times bigger as well.

## $600 \times 3=1800$

600 is 100 times bigger than 6, so I know that my answer must be at least 100 times bigger as well.

## Stage 2

Children should use partitioning and the Distributive Law when multiplying larger numbers to help them segment the digits into number facts that can be derived from times table knowledge. This will help the children to make the step towards Formal Short Multiplication and to underpin the children's understanding between number facts and the answers given in the different stages of Formal Multiplication.

## The Distributive Law

$$
\begin{array}{l|l}
\left\lvert\, \begin{array}{l}
\text { When multiplying multi-digit numbers } \\
\text { children should be taught and } \\
\text { encouraged to approach them using }
\end{array}\right. \\
\text { the Distributive Law. This states that } \\
\text { we can 'distribute' our multiplier } \\
\text { across each place value column to } \\
\text { solve the problem. }
\end{array}
$$

23 can be partitioned, or distributed, into 2 tens, or 20

## $20 \times 8=160$

The children should be able to recognise that $20 \times 8=$ ? is 10 times bigger than $2 \times 8=16$ and as such the answer should be 10 times bigger.

## $3 \times 8=24$

 through knowledge and practise of key number facts.
## $160+24=184$

By adding their two answers together the children should recognise and understand that they are able to come to a final answer to the problem.

## $23 \times 8=184$

## Stage 3

As the children become confident in both key number facts and the Distributive Law, they should move towards Formal Short Multiplication with 2 digit numbers.

|  | $\mathbf{2}$ | $\mathbf{1}$ | Children should be taught and <br> encouraged to set out their <br> calculations in the formal style with <br> the lager multiplicand above the <br> multiglier and with each number in <br> its correct place value column. |
| :--- | :--- | :--- | :--- |
| $\mathbf{1}$ | $\mathbf{6}$ | $\mathbf{8}$ |  |

The children should work with multipliers within the Multiplication Table groupings for Y 3 and multipliers no larger than 2 digits.

2
When needed, children should exchange any digits above the next digit in the corresponding place value column. For example, $4 \times 4=16$ so the 6 units will be placed within the total lines and the 1 , representing 10 , should be placed above the total line in the tens

## Children should also be able to:

- Recall and use Multiplication and Division facts for the 3, 4 and 8 Multiplication Tables.
- Solve problems, including missing number problems, involving Multiplication and Division relating to known facts.



## End of year objective:

Multiply 2 and $\mathbf{3}$ digit numbers by a 1 digit number using formal written layout (Short Multiplication).

Children should recap on the formal written methods taught at the end of Y 3 and expand upon these during their time in Y 4 .

| Children should be taught and encouraged <br> to exchange multiplied digits above the digit <br> in the next column. Children should be |
| :--- | :--- | :--- | :--- |
| expected to move towards exchanging |
| across multiple columns within Formal Short |
| Multiplication. |

Children should move towards multiplying 3 digit numbers by 1 digit numbers.

## 34 <br> 2

| $x$ | 2 | 1 | 7 |
| :--- | :--- | :--- | :--- |
| 2 | 3 | 9 | 4 |

## Children should also be able to:

- Recall Multiplication and Division facts up to $\mathbf{1 2 \times 1 2}$ (Statutory Requirement)

As part of the national curriculum, children are required to be confident in their knowledge of all key Multiplication Tables by the end of Y 4 .

- Solve Multiplication and Division problems in context.

Sarah's cat eats one tin of this cat food each day.


How much does it cost to feed Sarah's cat for 7 days?


## End of year objective:

> Multiply numbers up to $\mathbf{4}$ digits by a 1 or $\mathbf{2}$ digit number using a formal written method, including Long Multiplication for $\mathbf{2}$ digit numbers.

Children should extend their knowledge of Short Multiplication learnt in Y4 to Formal Long Multiplication. Children should move towards multiplying 3 and 4 digit numbers by a 2 digit multiplier.

Children should be taught and encouraged to exchange multiplied digits above the number in the next columns when needed.

When calculating using Formal Short Multiplication, children should be taught to exchange across multiple columns.


When calculating using Formal Long Multiplication children should be taught and encouraged to exchange multiplied digits above the number in the next column. When applicable, during the final addition stage of the calculation children should write any exchanged digits beneath the total line to help avoid miscalculation when totalling an answer.

## Children should also be able to:

- Identify Multiples and Factors, including finding all factor pairs of a number, and common factors of 2 numbers.
- Multiply and divide whole numbers and those involving decimals by 10, 100 and 1,000.
- Solve problems involving Multiplication and Division, including using their knowledge of Factors and Multiples, Squares and Cubes


## End of year objectives:

- Multiply multi-digit numbers, up to 4 digits by 2 digit whole numbers, using the formal written method of Long Multiplication.
- Multiply 1 digit numbers with up to 2d.p. by whole numbers using formal written methods where the answer has up to 2d.p.

Children should build upon the skills learned in Y5 to become confident in the multiplication of larger numbers.

|  | $\mathbf{2}$ | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{4}$ |
| :--- | :--- | :--- | :--- | :--- |
| x |  |  | $\mathbf{2}$ | 6 |
| $\mathbf{1}$ | 2 | 7 | 4 | 4 |
| 4 | 2 | 4 | 8 | 0 |
| 5 | 5 | 2 | 2 | 4 |

Children should be taught and encouraged to 'exchange' multiplied digits above the number in the next column. For example $6 \times 4$ $=24$ so the 4 would be written into the units column and the 2 , representing 20 , would be written above the next column. The 'exchanged' 2 would then be added to the product of 6 and 2 .

When multiplying decimals by whole numbers children should be taught and encouraged to write each number to the same number of decimal places through identification. For example, 4 units have the same value as 400 hundredths.

When finding the final answer to the calculation, the children should be taught and encouraged to write any 'exchanged' digits from their addition underneath the total line. This will help to avoid miscalculation when totalling an answer.

When multiplying by two digit numbers, children should be taught and encouraged to recognise that the second stage of the calculation takes place with numbers that are ten times larger and thus ' 0 ' should be introduced as a place holder.

Children should be taught to recognise that whenever a number is multiplied by 0 the answer will always be 0 and as such, they can increase the efficiency of their calculation by viewing these digits as place holders within the layout of the formal method.

## Children should also be able to:

- Identify Common Factors, Common Multiples and Prime Numbers
- Use estimation to check that an answer is correct within the context of a problem.
- Solve multi-step problems in context that incorporate Multiplication as at least one of the steps.



## Early learning Goal:

Explore and represent patterns with numbers up to 10, including how quantities can be distributed equally.

Children should be encouraged to develop their own mental picture surrounding numbers in preparation for calculation. They should be given a wide range of opportunities to experiment with Division and sharing in practical scenarios, e.g. role play, counting games and small world play. Children should be given access to a wide range of mathematical resources for counting so that they are aware of them and how they can be used later in the school. Children should begin to identify their own mathematical problems based on their own interest and fascinations using the resources and equipment made available to them.

Children should also be given the opportunity to experiment with various 'real world' objects that show Division and sharing through the use of non-formal groupings and arrays.


Children should also be encouraged to develop their own ways of recording 'real world' Division through drawings and jottings.


The picture shows a child's drawing to solve the problem, 'Can 8 be shared equally between 2?' The child has contextualised the problem through birds and eggs, sharing and linking 4 eggs to each bird.


Further information and guidance for the teaching of Division should be taken from the Numicon Firm Foundations Teaching Guide.

## End of year objective:

Solve one-step problems involving Division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

Children should continue to use a wide range of practical equipment as a mean of solving Division problems with the support of the teacher. They should use a wide variety of 'real world' objects that show Division as an array, such as egg boxes, ice cube trays, cake tins and chocolate bars. In Year One, children should begin to approach one step problems using concrete resources and pictorial representations to underpin their learning. Children should begin to recognise Division as the opposite of Multiplication as a means for solving problems.

Four children share these shells.

They each get the same number of shells.


How many shells does each child get?


How many sets of 5 stars can you make?

Children should continue to approach Division problems, such as the one above, through 'grouping' or 'sharing'. They should be introduced to remainders and begin to describe them as 'left over' and identify that Division does not always leave you with fully equal groups.

Children should also continue to use Base 10 and Numicon equipment. Further in depth guidance can be found in the Numicon Kit 1 Handbook.


Calculate mathematical statements for Division within the Multiplication Tables and write them using the Division ( $\div$ ) and Equals ( $=$ ) signs.

Children should be taught and encouraged to use a range of practical equipment and jottings to approach Division as grouping or repeated Subtraction.

$$
30 \div 5=6
$$

Children should be taught and encouraged to recognise that the question above is asking them 'how many equal groups of 5 are in 30 ?' Division problems can be approached using similar apparatus to Multiplication as they are inverse.

This problem could be approached with counters, the children counting out 30 counters before dividing them into equal groups of 5 until no more counters remain as is shown below. The children should be encouraged to arrange them in an organised pattern to aid calculation accuracy.

## 00000000000000000000000000000

Children should also be taught and encouraged to divide numbers that leave a remainder in this way:

## $21 \div 5=4$ remainder 1



## Children should also be able to:

- Solve problems involving Multiplication and Division, using materials, arrays, repeated Addition, mental methods, and Multiplication and Division facts, including problems in contexts.
- Recall and use Multiplication and Division facts for the 2,5 and $\mathbf{1 0}$ Multiplication Tables, including recognising odd and even numbers


## End of year objective:

To practise and recall mental methods for Division, progressing towards formal written methods for Division.

## Stage 1

Children should continue to build upon knowledge of grouping from Y2 and begin to apply this to increasingly larger numbers both in and out of context. From this point children should be introduced to inverse operations as a means of deriving number facts.
$4 \times 8=32$ and so $32 \div 4=8$ and $32 \div 8=4$
This problem can be explained using an array:

We can see that 32 is made up of 4 equal groups of 8 .

Children should be taught and encouraged to write this using the appropriate symbols, in this case $32 \div 4=8$ and $32 \div 8=4$.


## Stage 2

Over the course of Y3, children should rehearse mental methods for Division, starting within Multiplication Tables facts and applying this skill to larger related numbers e.g. 800 $\div \mathbf{4 = 2 0 0}$ can be derived from $80 \div 4=20$ and $8 \div \mathbf{4 = 2}$.

In order to solve this problem the children should be taught and encouraged to use place value skills to recognise the relationship between 800 and 8 and what effect this has upon the answer.

$$
8 \div 4=2
$$

## $80 \div 4=20$

80 is 10 times bigger than my route calculation number so I know my answer will be at least 10 times bigger because of this.

## $800 \div 4=200$

800 is 100 times bigger than my route calculation number so I know my answer will be at least 100 times bigger because of this.

## Stage 3

Children should continue to practise derived Division facts from known Multiplication Tables. Alongside this children should be introduced to the Formal Short Division layout for Multiplication Tables facts. Within Formal Short Division Y3 children will not be expected to have final remainders.


## Children should also be able to:

- Recall and use Multiplication and Division facts for the 3, 4 and 8 Multiplication Tables.
- Solve problems, including missing number problems, involving Multiplication and Division relating to known facts.


## End of year objective:

To use formal written methods (Short Division) to divide numbers without remainders.

Over the course of Y 4 children should rehearse mental methods for Division, starting within Multiplication Tables facts and applying this skill to larger related numbers, e.g. $\mathbf{6 0 0} \div \mathbf{3 = \mathbf { 2 0 0 }}$ can be
 Division to approach this problem by the end of Y 4 .

## 3

Moving on from derived facts, children should be introduced to Formal Short Division, with 2 digit numbers. They should be taught and encouraged to write the number of groups above the total line.

When the children are confident at setting out their work in this way, they should begin to use Formal Short Division with larger numbers, no greater than 3 digit whole numbers. All answers at this stage should be whole numbers and children should become confident at this stage before remainders are introduced.

## 314 <br> 3

For example, there is 1 group of 3 in 4 with 1 remainder. The answer, 1 , is written above the total line and the remainder of 1 is then exchanged over to the next column and we see how many groups of 3 are in 12 before writing the answer above.

By the end of Y4, the children should be able to use Formal Short Division with 3 digit numbers to produce whole number answers. Within the calculation the children should be taught to exchange any remaining numbers over to the next column and include this within their calculation.

## Children should also be able to:

- Recall Multiplication and Division facts up to $12 \times 12$
- Children should practise mental methods for Division facts, e.g. $600 \div 3=200$ can be derived from $60 \div 3=20$
- Solve Multiplication and Division problems in context.


## End of year objective:

Divide numbers up to 4 digits by a 1 digit number using the formal written method of Short Division and interpret remainders appropriately for the context.

Children should be taught to give remainders to Short Division calculations in a variety of formats, including fractions, rounded numbers and decimals. This should then be applied to a problem solving context where appropriate.


Rounding answers in context:

## A box holds 6 eggs.



How many boxes are needed to hold 52 eggs?

To solve this problem the children would need to calculate $52 \div 6=8 \frac{2}{3}$. From this answer, they should be taught and encouraged to recognise that they must round this number to the nearest whole egg box in order to solve the problem successfully.


When dividing to give a decimal remainder, children should be taught and encouraged to exchange any 'final remainders' across the decimal point and continue the calculation in order to give an answer as a decimal.

## Children should also be able to:

- Multiply and Divide whole numbers and those involving decimals by 10, 100 and 1,000.
- Solve problems involving Multiplication and Division, including using their knowledge of Factors and Multiples, Squares and Cubes.


## End of year objectives:

- Divide numbers of up to 4 digits by a 2 digit whole number using the formal written method of Long Division and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context.
- Divide numbers by up to 4 digits by a 1 digit number using the formal written method of Short Division where appropriate, interpreting remainders according to the context.

Children should build upon the knowledge of Short Division gained within Y5 and apply this skill to Formal Long Division with increasingly larger numbers.


The children should also be able to use and apply Formal Short Division when calculating with smaller numbers.
5

| 0 | 9 | 0 | 6 |
| :---: | :---: | :---: | :---: |
| $A$ | ${ }^{4} 5$ | $\not \partial$ | ${ }^{3} 2$ |

Within both elements of formal Division children should be taught and encouraged to use estimation and inverse operations to check the accuracy of their answer according to the context.

As with Long Division, when a remainder is present Children should write this as appropriate for the context of the problem. When solving a pure calculation, this should be in the form of a reduced fraction.

Primary School
Laying the Foundations for Children to Thrive

## Teaching Fractions

## Concrete, Pictorial and Abstract approach

Concrete and pictorial representations of mathematics are chosen carefully to help build procedural and conceptual knowledge together.

Concrete - the "doing" stage, using concrete objects to model problems.

Pictorial - the "seeing" stage, using representations of the objects to model problems.
Building or drawing a model makes it easier for children to grasp concepts they traditionally find more difficult as it helps them visualise the problem and make it more accessible.


#### Abstract

- the "symbolic" stage, where children are able to use abstract symbols to model problems


Although CPA have been presented as three distinct stages, a skilled teacher will go back and forth between each representation to reinforce concepts.
'Used well, manipulatives can enable pupils to inquire themselves- becoming independent learners and thinkers. They can also provide a common language with which to communicate cognitive models for abstract ideas.'

Drury, H. (2015)

Throughout all fraction work the following vocabulary should be referred to and used by teachers and children.

## Part Whole

## Numerator <br> Denominator

Equal parts of a whole must be reinforced. The whole could be a shape, a length a set of objects, a number etc. Children must experience fractions in many different contexts.

## Reception

I can solve problems involving distributing quantities equally.

Concrete: Throughout Reception children should experience what a half looks like in a wide range of contexts and what a half doesn't look like. The half of something needs to be explored in many concrete situations.


## Year 1

Recognise, find and name a half as one of two equal parts and a quarter as one of four equal parts of an object, shape or quantity

Concrete: Children should build on the concrete examples used in Reception to further their understanding of a half. Children should continue to experience what a half looks like in a wide range of contexts and also what a half doesn't look like. Children must have a secure understanding that a half is two equal parts of a whole and a quarter is four equal parts of a whole through a variety of concrete examples.


The pencil is half the length of the piece of paper

Sorting objects


## Pictorial:

Children use sorting circles to find half and a quarter of quantities


Half
Quarter

Children then progress onto a bar model


## Year 2

Recognise, name, write and find fractions $1 / 3 ; 1 / 4 ; 1 / 2$ and $3 / 4$ of a length, shape, set of objects or quantity.

Concrete: Children continue to build on concrete models used in Year 1 to further their understanding of fractions. Children must recognise and have a secure understanding that a third is three equal parts of a whole and three quarters is the value of $\mathbf{3}$ equal parts of the 4 equal parts that makes up the whole.


Pictorial: Children should draw diagrams of fractions.


Children use sorting circles to find a third and three quarters of a quantity


Third Three Quarters

Children then progress onto a bar model


## Recognise the equivalence of $2 / 4$ and $1 / 2$

## Concrete:

Practical activities, using the same size strips of paper can support with the concept of 'equivalence'. Strips rather than circles are best to use when comparing because they can be placed above/below each other but children should not only see fractions in a rectangle.


Pictorial: Children draw equivalent fractions


## Year 3

Recognise and show, using diagrams, equivalent fractions for halves, quarters, thirds, fifths, sixths and eighths

Concrete: Building on Year 2 practical examples using the same size strips of paper can support with the concept of 'equivalence'. Strips rather than circles are best to use when comparing because they can be placed above/below each other but children should not only see fractions in a rectangle.


## Pictorial:



## Recognise fractions as a division of a quantity

Concrete: Building on Year 2 and 3, children should use cubes and Cuisenaire rods to provide a concrete example of the concept. Extend this in Year 4 to using numicon e.g 1/5 of 20


Pictorial: Building on year 2, children should use bar model as a pictorial representation. E.g 1/6 of 24

| 4 | 4 | 4 | 4 | 4 | 4 |
| :--- | :--- | :--- | :--- | :--- | :--- |

## Concrete:

Use numicon as a concrete representation e.g 2/6+3/6=

$5 / 7-2 / 7=$

Use numicon as a concrete representation e.g 5/7-2/7 =


## Pictorial:

Use bar model as a pictorial representation e.g. 2/6 + 3/6 =


Recognise fractions as a division of a quantity
$5 / 7-2 / 7=$


## Year 4

## Recognise and show, using diagrams, families of common equivalent fractions

Concrete: If required, use the concrete representations from Year 3.

## Pictorial:

Continue to build on the pictorial representations in Year 3 and extend to include


Solve problems involving increasingly harder fractions to calculate quantities, and fractions to divide quantities, including non-unit fractions where the answer is a whole number.

Concrete: If required use concrete representations from Year 3.

Pictorial: Building on the bar model used in Year 3. E.g. 2/5 of 45


Add and subtract fractions with the same denominator, including improper fractions

## Concrete:

If required, use the concrete representations from Years 3 for adding and subtracting fractions less than zero. 5/6 + 3/6 =


## Pictorial:

Using a number line: E.g. $2 / 5+4 / 5=11 / 5$


## Year 5

Compare and order fractions with denominators that are all multiples

Concrete: If required, use the concrete examples in Years 3 and 4 to develop children's understanding of equivalent fractions.

## Pictorial:

Use arrays. E.g Which is bigger $1 / 4$ or $2 / 5$ ?


Recognise mixed numbers and improper fractions and convert from one to the other

Concrete: Use numicon. For example, $11 / 6=7 / 6$


Pictorial: Use a bar model to convert between fractions.

## Add and subtract fractions with denominators that are all multiples

Concrete: If required, use the concrete examples in Years 3 and 4 to develop children's understanding of adding and subtracting fractions with same denominators.

Pictorial: If required, use the pictorial examples in Years 3 and 4 to develop children's understanding of adding and subtracting fractions with same denominators.

Array. For example, $1 / 3$ and $2 / 4=10 / 12=5 / 6$


Concrete: Use numicon, for example $2 / 3 \times 3=$


Pictorial: Use bar method, for example $2 / 3 \times 3=$

thirds

## Year 6

Compare and order fractions including fractions > 1

## Concrete and Pictorial:

If required, use the concrete and pictorial models to compare fractions up to 1 . Use abstract approach for fractions greater than 1.

Add and subtract fractions with different denominators and mixed numbers

## Concrete and Pictorial:

If required, use the concrete and pictorial models to add fractions with denominators that are multiples from Year 5, then use abstract approach for different denominators and mixed numbers.

Multiply simple pairs of proper fractions, writing the answer in its simplest form

## Pictorial:

Arrays, for example: $1 / 3 \times 1 / 2=$


Divide proper fractions by whole numbers ( $1 / 8 \div 2=1 / 16$ )

## Concrete:

Use fraction strips to create a concrete representation, for example $1 / 3 \div 2=$


## Pictorial:

Children use a diagram to represent the above.

# Maths Vocabulary Key Stage Two (also in KS2 Pupil Planner) 

| Year 3 | Year 4 <br> (all Y3 vocabulary plus the following-) |
| :---: | :---: |
| Number - Place Value | Number - Place Value |
| 2-digit, base 10, pattern, sequence, hundred, partition, recombine, thousand, 3 -digit, ascending, descending | Roman numerals, round, nearest, approximately, negative, minus, tenths, hundredths |
| Number - Addition \& Subtraction | Number - Addition \& Subtraction |
| Bar model, operation, inverse operation, column, exchange, bridge, method, column addition and subtraction, regroup, efficient, estimate. | Formal methods |
| Number - Multiplication \& Division | Number - Multiplication \& Division |
| Times-table, facts, multiples, repeated addition, lots of, multiply, multiplied by, times, commutative, array, go into, divide, divide between, division, dividing, grouping, sharing, remainder, divisor, dividend, quotient. | Product, associative law, commutativity, factor, factor pair. |
| Number - Fractions | Number - Fractions |
| unit fraction, numerator, denominator, equivalence, equivalent, non-unit fraction, decimal, decimal point. | Proper fraction, improper fraction, mixed number |
| Measurement | Measurement |
| Change, total, distance; metres, $\mathrm{g} / \mathrm{kg}$, $\mathrm{ml} / \mathrm{l}$, temperature, thermometer, degrees Celsius, increase, decrease, warmer, colder, quarter past/to, start, duration, end, interval, $\mathrm{cm}, \mathrm{mm}$, perimeter, leap year, minutes past/to; a.m., p.m., analogue, digital, twelve-hour /twenty-four- hour clock. | km , rectilinear, area, cm squared, square cm , warmest, coldest. |
| Geometry - Shape, Position \& Direction | Geometry - Shape, Position \& Direction |
| Pentagon, hexagon, octagon, quadrilateral, prism, vertices, vertex; rotate, symmetry, symmetrical, line of symmetry; horizontal, vertical, fold, pattern, repeating pattern. direction, forwards, backwards, right angle, rotation, clockwise, anticlockwise, parallel, perpendicular, surface, acute angle, obtuse angle, North, South, East, West. | Isosceles, scalene, equilateral, rhombus, parallelogram, trapezium; regular polygon, mirror line, reflect, coordinates, translation, first quadrant, $x$-axis, $y$-axis. |
| Data Handling | Data Handling |
| Count, tally, tally chart, table, data, represent, sort, pictogram, symbol, block diagram, axis, label, title, scale, most popular, most common, least popular, least common, Venn diagram, Carrol diagram, bar chart, frequency table. | Continuous data, discrete data, line graph, $x$-axis, $y$-axis. |


| Year 5 <br> (all Y3, Y4 vocabulary plus the following-) | Year 6 <br> (all Y3, Y4, Y5 vocabulary plus the following-) |
| :---: | :---: |
| Number - Place Value | Number - Place Value |
| Numbers to a million; Roman numerals to one thousand; powers of 10. | Algebra: function, input, output, algebra, algebraic, rule, expression, substitute, formula, formulae, equation, value, possible values, enumerate. |
| Number - Addition \& Subtraction | Number - Addition \& Subtraction |
| Place holder. |  |
| Number - Multiplication \& Division | Number - Multiplication \& Division |
| Common factor, prime number, composite number, prime factor, square number, cubed number, round up/down. | Order of operations, BIDMAS, common multiple, lowest common multiple. |
| Number - Fractions | Number - Fractions |
| Common denominator, thousandth, simplify, simplified, convert, per cent, percentage, per hundred. | Cancel, highest common factor, common denominator, ratio, proportion, enlargement, scale factor. |
| Measurement | Measurement |
| Imperial units, metric units, inches, lbs, pints, timetable, compound shape, volume, capacity, cm cubed/cubic cm. | Tonnes, ounces, stone, miles, vertically opposite (angles), internal angles, circumference, radius, diameter, centre. |
| Geometry - Shape, Position \& Direction | Geometry - Shape, Position \& Direction |
| Degrees, protractor, reflex angle, irregular polygon, dimensions, net, reflection, reflect. | Four quadrants. |
| Data Handling | Data Handling |
|  | mean, pie chart, minimum, maximum, range |


| Addition | Subtraction | Multiplication | Division |
| :---: | :---: | :---: | :---: |
| add | subtract | multiply | divide |
| plus | minus | times | divided by |
| and | take | lots of | division |
| more | take away | groups of | share |
| altogether | difference | product | shared by |
| total | less |  | split |
| sum of | decrease |  | half |
| increase | deficit |  |  |
| addition | reduce |  |  |

